Load Sequence Interaction Effects in Structural Durability

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Introduction

constant amplitude

variable amplitude

load

cycles

N, $\bar{N}$ [log]
Fatigue damage evolution

Physical measure of damage

Affine damage growth curves

$D = \sum \frac{n_i}{N_i} = 1$
Integration of a crack growth law

\[ \frac{da}{dn} = C \cdot \Delta K^m \]

\[ \Delta K = \Delta \sigma \sqrt{\pi a} \cdot Y(a, G) \]

\[ \int_{a_{j-1}}^{a_j} \frac{da}{(\sqrt{a} \cdot Y(a, G))^m} = C_j \cdot \Delta \sigma_j^m \cdot (\sqrt{\pi})^m \cdot n_j \]

- High
- Medium
- Low

Crack length \( a \)

At failure \( a_e \)

Start \( a_0 \)

Cycles \( n, N_j \)
crack growth curves from Paris-type laws are affine

\[
\int_{a_0}^{a_f} \frac{da}{a \left( \sqrt{a} \cdot Y(a,G) \right)^m} = \frac{n_j}{N_j}
\]

linear damage accumulation hypothesis

\[
\sum \frac{n_j}{N_j} = 1
\]

Such Miner-type life prediction results do not depend on the load sequence
Origins of load interaction effects

- Plasticity
- Geometrical alterations

May be subject to revision with increasing scientific insight
Fatigue strength expressed in terms of life curves and stress lines

life curves for $R = \text{const.}$

stress lines for $N = \text{const.}$

Haigh-Diagram

$$\sigma_m = \frac{1+R}{1-R} \cdot \sigma_a$$
Type 1 sequence effects due to mean stress rearrangement

Solve the problem by using an appropriate version of the local strain approach.
Mean stress effects in the crack growth regime

Crack growth rates for S460N various R-rates ($R = \sigma_{\text{min}}/\sigma_{\text{max}}$)
Crack closure, experimental observation

![Diagram showing crack closure and load vs. near tip crack flank displacement.]

- **Fatigue crack closure**
- **Load**
- **Near tip crack flank displacement**
Closure-free crack growth rates for S460N

\[ \Delta K = K_{\text{max}} - K_{\text{op}} = \Delta K_{\text{eff}} \quad [\text{MPa}\sqrt{\text{m}}] \]
Origin of crack closure

- Plasticity
- Oxides
- Roughness
- Penetrating media
- Martensite transformation
Empirical equations

\[
\frac{da}{dn} = C \left( \frac{U \cdot \Delta K}{\Delta K_{\text{eff}}} \right)^m ; \quad U = (0.5 + 0.4R)
\]

\[
\frac{da}{dn} = C \left( \frac{U \cdot \Delta K}{\Delta K_{\text{eff}}} \right)^m ; \quad U = (0.55 + 0.33R + 0.12R)
\]

\[
\frac{da}{dn} = C \left( \frac{1 - f}{1 - R} \cdot \Delta K \right)^m \left( 1 - \frac{\Delta K_{\text{th}}}{\Delta K} \right)^p \left( 1 - \frac{K_{\text{max}}}{K_c} \right)^q
\]

\[
f = \begin{cases} 
(A_0 + A_1 \cdot R + A_2 \cdot R^2 + A_3 \cdot R^3) & \text{for } R > 0 \\
(A_0 + A_1 \cdot R) & \text{for } R \leq 0
\end{cases}
\]

\[
A_0 = 0.535 \cos \left( \frac{\pi \cdot \sigma_{\text{max}}}{2 \cdot \sigma_F} \right) \quad A_1 = 0.344 \cdot \frac{\sigma_{\text{max}}}{\sigma_F}
\]

\[
A_3 = 2 \cdot A_0 + A_1 - 1 \quad A_2 = 1 - A_0 - A_1 - A_3
\]
Extended Dugdale-Barenblatt Model acc. to Seeger, Führing, Newman

stress distributions

contact stresses
Simulation for initial overload followed by constant amplitude cycling
Explanation of sequence effects

load sequence

max min = 0

(max)

(min)

flattening by contact

crack tip

X

crack opening
Type 2a sequence effects; long cracks
Short cracks
Crack opening strains rather than stress

\[ \varepsilon_{\text{tinygage}} \]

\[ \varepsilon_{\text{op}} \]

\[ \sigma \]

\[ \varepsilon_a = 0.5\% \]

Life \( N = 7700 \) Ssp.

\[ \varepsilon_a = 0.2\% \]

\( N = 200000 \) Ssp.
**Type 2b sequence effects; short cracks**

- **$a=0.5\text{mm}$**
- **$a=1\text{mm}$**
- **$100\text{MPa}$**
- **$0.1\%$**

**Graph:**
- Life in Experiment: $N = 26500 \text{ cyc}$
- Life calculated with Miner's rule: $N = 27/(1/7700 + 26/200000) = 104000 \text{ cyc}$
Simple model
Crack opening stresses

\[ \frac{\sigma_{op}}{\sigma_F} \]

\[ \frac{\sigma_{max}}{\sigma_F} \]

R = -1

- Newman
- AlMg4.5Mn
- S460N
- A36
- 10 Cr Mo 9 10
- AISI 4340
Model performance dealing with a random sequence

a) frequency distribution of reversal point strains

- Upper reversal points
- Lower reversal points

b) crack opening strains

- Scatter band of experimental data (dimension of strain gage: 0.25 mm x 0.2 mm)
- Occurrence of the largest cycle

- AIMg4.5Mn
- Predicted by the proposed algorithm

- Crack length $2a = 0.25$ mm
- Crack length $2a = 0.50$ mm
Comparison experiment-prediction, Gaussian distributions, $I = 0.99$, $R = -1$, AlMg4.5Mn

$H_0 = 10^4$, $\bar{R} = -1$

$H_0 = 5 \cdot 10^5$, $\bar{R} = -1$

notch strain simulation $K_i = 2.5$

AlMg4.5Mn
Cyclic $J$-integral for short cracks as crack driving force

For
$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}}$$

there is
$$J = \left(1.24 \cdot \frac{\sigma^2}{E} + \frac{1.02}{\sqrt{n'}} \cdot \sigma \cdot \varepsilon_p\right) \cdot a$$

$$\Delta J_{\text{eff}} = \left(1.24 \left(\frac{\sigma_{\text{max}} - \sigma_{\text{cl}}}{E}\right)^2 + \frac{1.02}{\sqrt{n'}} \left(\sigma_{\text{max}} - \sigma_{\text{cl}}\right) \cdot \left(\varepsilon_{\text{max}} - \varepsilon_{\text{cl}}\right) - \frac{\sigma_{\text{max}} - \sigma_{\text{cl}}}{E}\right) \cdot a$$

Integration of
$$\frac{da}{dn} = C \left(\Delta J_{\text{eff}}\right)^m$$
is equivalent to a Miner-type damage accumulation in terms of $aP_j$-$N$ curve
Fatigue crack growth, approaching monotonic strength, threshold and endurance limit

**Paris-law**

\[
\frac{da}{dn} = C \cdot \Delta K^m
\]

- Region 1: Threshold \( \Delta K_{th} \)
- Region 2: Paris-law
- Region 3: Endurance limit \( \Delta K_c \)

**Type 3 sequence effects due to interference of monotonic failure modes:**
- non-power crack growth laws yield non-affine crack growth curves
- failure condition „fracture“ leads to stress-dependent final crack lengths

Usually, this sequence effect plays a minor role.
Relation between crack length, endurance limit and damage sum

Type 4 sequence effect due to decreasing fatigue limit. Cycles with amplitudes smaller than the constant amplitude fatigue limit cause damage (FCG) if applied after some pre-damage due to large amplitude cycles. Usually, this sequence effect plays a major role.
Considering decreasing endurance limit in a damage accumulation rule

\[ x_i \cdot \sum_{j=1}^{i_e} \frac{h_j}{N_j} = D_i \]

\[ x_i = \frac{D_i(\Delta\sigma_{jE+1}) - D_i(\Delta\sigma_{jE})}{\sum_{j=1}^{i_e} \frac{h_j}{N_j}} \]

\[ \bar{n}_i = H_0 \frac{D_i(\Delta\sigma_{jE+1}) - D_i(\Delta\sigma_{jE})}{\sum_{j=1}^{i_e} \frac{h_j}{N_j}} \]

\[ \bar{n}_2 = H_0 \frac{D_i(\Delta\sigma_{jE+2}) - D_i(\Delta\sigma_{jE+1})}{\sum_{j=1}^{i_{e+1}} \frac{h_j}{N_j}} \]

\[ \bar{n}_3 = H_0 \frac{D_i(\Delta\sigma_{jE+3}) - D_i(\Delta\sigma_{jE+2})}{\sum_{j=1}^{i_{e+2}} \frac{h_j}{N_j}} \]

\[ N = H_0 \sum_{i=1}^{i_e} \frac{D_i(\Delta\sigma_{jE+i}) - D_i(\Delta\sigma_{jE+i-1})}{\sum_{j=1}^{i_{e+i-1}} \frac{h_j}{N_j}} \]
German FKM-guideline rule for assessing damage of small cycles: „Consequent Miner‘s rule“

\[
\frac{S}{S_{E,0}} = \left( \frac{N}{N_{E,0}} \right)^{1/k}
\]

\[
\frac{S_{E}}{S_{E,0}} = (1 - D)^{\frac{1}{k-1}}
\]

\[
D = \sum \frac{n_i}{N_i}
\]

stress-life curve

Nominal stress

\(S_{E,0}\)

\(N_{E,0}\)  \(N\ [\log]\)

\(D_1\)  \(D_2\)  \(D_3\)  1
Comparison of Miner’s rules variants

\[ \Delta \sigma, \Delta \sigma_{\text{max}} \, [\text{MPa}] \]

\[ C=5 \times 10^{-9} \frac{\text{mm}}{\text{Ssp.}} (\text{MPa} \sqrt{\text{m}})^{-3} \]

\[ \Delta K_{\text{th}} = 5 \text{MPa} \sqrt{\text{m}} \]

- elementary
- modified
- consequently
- original

\[ \Delta \sigma_j / \Delta \sigma_1 \]

\[ \text{cumul. frequency } H \]

\[ N, \bar{N} \]
Comparison experiment-prediction, Gaussian distributions, $I = 0.99$, $R = -1$, AlMg4.5Mn
Omission of small cycles discloses their contribution to fatigue damage

Normalised filter level

\[ \frac{P_{\text{SWT,Filter}}}{P_{\text{SWT,E,0}}} \]

notch strain simulation

- \( K_t = 2.5 \)
- AlMg4.5Mn
- Gaussian
- \( S_a = 108 \text{ MPa} \)
- \( H_0 = 5 \cdot 10^5 \)
- \( R = -1; I = 0.99 \)

Runs through the spectrum until failure (0.5mm surface crack initiation)
Omission of small cycles discloses their contribution to fatigue damage
Damage contribution of omitted cycles

- Straight line distribution
- StE 690
- Notch strain simulation
- \( K_t = 2.5 \)
- Neuber

\[ P_{Fi} \quad [N/mm^2] \]

\[ \begin{array}{c}
0.47 \cdot P_{10^7} \\
0.70 \cdot P_{10^7} \\
450 \\
400 \\
300 \\
200 \\
0 \\
\end{array} \]

\[ \Delta D_{OMS} [\%] \]
Variable amplitude life:
Two-level testing of autofrettaged specimens

\[ \Delta p = 1500 \text{bar} \quad \text{factor 1001} \]
\[ \Delta p = 1125 \text{bar} \quad 2 \]
\[ \Delta p = 800 \text{bar} \quad 2.5 \]

- constant amplitude; \( R=0 \)
- constant amplitude; \( R=0.5 \)
- two-level; 1:1000; \( p_{\text{max}}=3000\text{bar} \)
- two-level; 1:1000; \( p_{\text{max}}=2250\text{bar} \)
- two-level; 1:10000; \( p_{\text{max}}=2250\text{bar} \)
Life estimates based on various methods; Two-level-tests

![Graph showing life estimates](image)

- **Experimental**
- **LA**
- **LEFM M2**
- **LA + LEFM M2**

The graph compares life estimates using different methods. The x-axis represents the number of cycles (N), while the y-axis shows the stress range (Δp [bar]). The legend indicates the different methods used for estimation.
Micro crack growth acc. to Tokaji and Ogawa

Medium carbon steel (S45C)

$\sigma = 240 \text{ MPa}$

$R = -1$

surface crack length $c$ (mm)
Model acc. to Tanaka

grain boundary crack slip band

idealised micro structure

mechanics model

yield stress distribution
Crack growth passing grain boundary

\[
\frac{da}{dn} = \left( 0.63 \cdot \frac{\delta}{\text{mm}} - 5.7 \cdot 10^{-5} \right)^{1.5}
\]

\[\sigma = 700 \text{ MPa} \]
\[\sigma_{F_1} = 350 \text{ MPa} \]
\[\sigma_{F_2} = 800 \text{ MPa} \]
\[\sigma = 469 \text{ MPa} \]

F1 = 350 MPa
F2 = 800 MPa

\[\frac{n}{N} = \left( 0.63 \frac{\delta}{\text{mm}} - 5.7 \cdot 10^{-5} \right)^{1.5}\]
Type 5 sequence effect due to (micro structural) inhomogeneities

- \( \frac{n_1}{N_1} = 0.6 \)
- \( \frac{n_2}{N_2} = 0.6 \)
- \( \sum \frac{n_i}{N_i} = 1.2 \)

- \( \frac{n_1}{N_1} = 0.4 \)
- \( \frac{n_2}{N_2} = 0.4 \)
- \( \sum \frac{n_i}{N_i} = 0.8 \)
## Summary

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<th>Type 2: Crack closure</th>
<th>Type 3: Monotonic failure</th>
<th>Type 4: Decreasing fatigue limit</th>
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- **Type 1:** Macroscopic mean stress rearrangement
- **Type 2:** Crack closure
  - Short cracks
  - Long cracks
- **Type 3:** Monotonic failure
- **Type 4:** Decreasing fatigue limit
- **Type 5:** Inhomogeneities