STRUREL in WOLAXIM Project

STRUREL is a set of programs for reliability analysis of structural, operational & other systems employing state-of-the-art techniques.

STATREL: A program for reliability oriented statistical analysis and simulation

COMREL: A program for time-invariant and time-variant component reliability analysis

SYSREL: A program for system reliability analysis including reliability updating

COSTREL: A program for reliability oriented optimisation

PERMAS-RA:
A multipurpose FE-Program (by INTES) coupled with **Comrel** & **Costrel**

COMREL-TI

COMREL-TI covers **T**ime-**I**nvariant reliability analysis of individual failure modes.

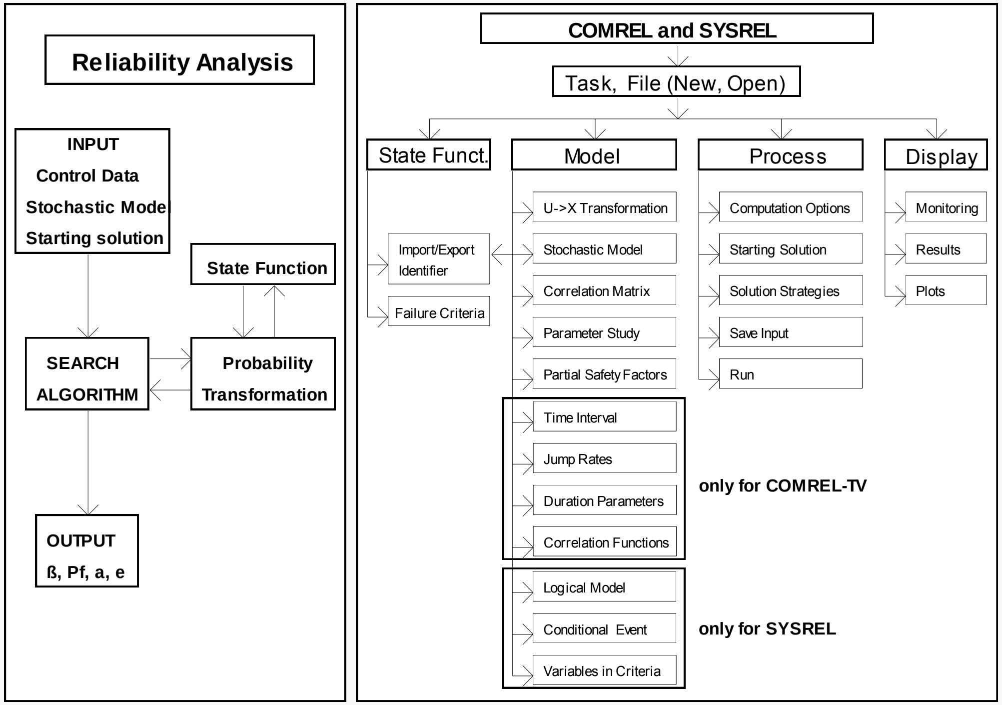
- advanced FORM/SORM methodology (sensitivity measures !)
- several algorithms to find the most likely failure point (β-point)
- a gradient free algorithm for non-differentiable failure criteria
- Importance Sampling schemes on top of FORM/SORM
- Monte Carlo simulation, Adaptive Sampling, Spherical Sampling
- arbitrary dependence structures in the stochastic model (Rosenblatt, Hermite and Nataf-models)

SYSREL

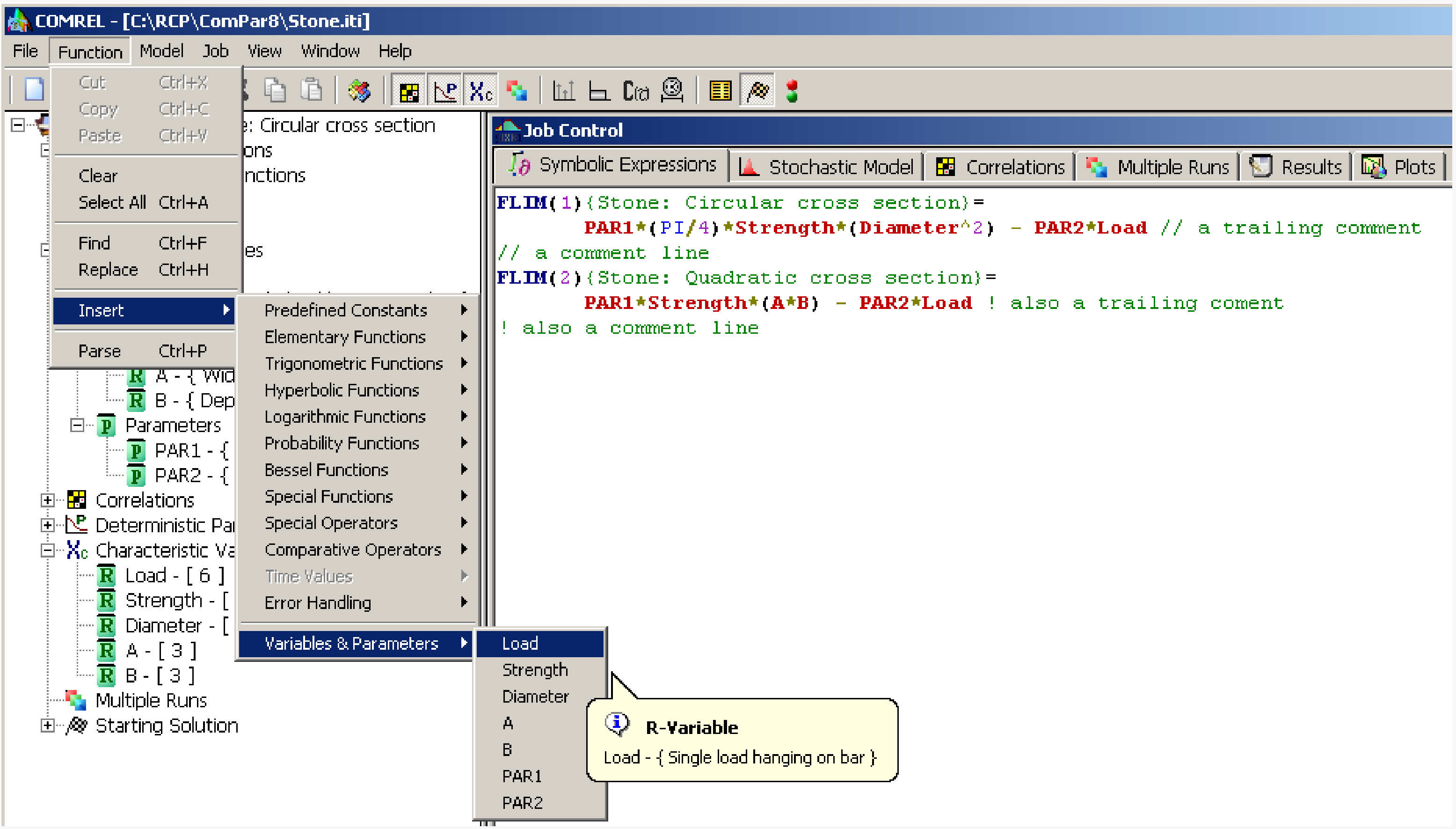
SYSREL for system reliability evaluation including event updating.

- FORM/SORM methodology
- two well-tested algorithms to find individual and joint β-points
- series system (union of failure events)
- parallel system (intersection of failure events)
- (minimal) set of parallel systems in series – minimal cut set
- <u>conditional events (observations, event updating)</u>
- stochastic modelling features (dependencies) as in **Comrel**
- sensitivity measures as in **Comrel**
- Monte Carlo simulation

Basic Data Flow and Sequence of Tasks



State Function Window and Model Options in *Comrel*

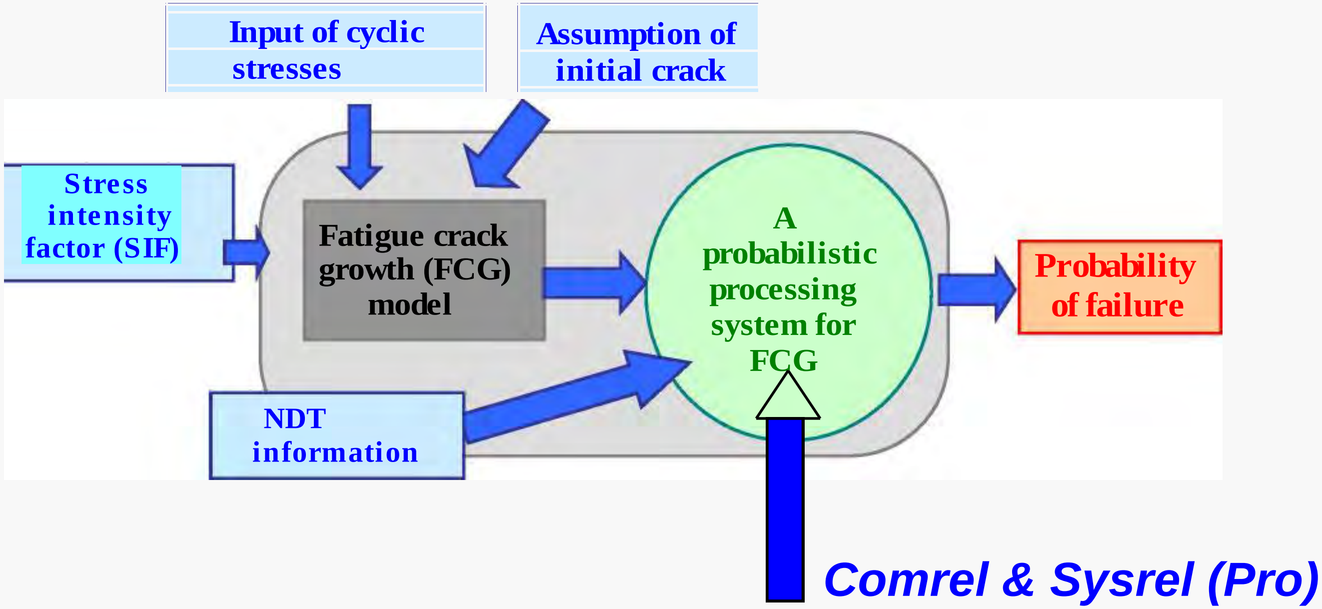


STRUREL

Pro-Versions with Fortran Interface

- ***Pro*** version for complicated iterative state functions and if external programs have to be linked to state functions
- **Comrel**, **Sysrel** & **Costrel** as static libraries in Fortran 77/90
- documentation of interfaces and parts of source code available as so called ***Toolkits***
- state functions to be written in Fortran 77/90 (… compile & link)
- Project: Corrosion assessment in pipelines (2001 – 2005)
  Sysrel-*Pro* for GE PS Oil & Gas, PII Pipeline Solutions, UK
- Project: Probabilistic crack growth (high cycle fatigue), (2007)
  Sysrel-*Pro* for TWI, Structural Integrity Technology Group, UK
- FP-7 Project: Whole Life Rail Axle Assessment and Improvement (2010)

WOLAXIM Project: *Comrel* and *Sysrel* to compute P_f



The NASGRO Model

$$\frac{da}{dN} = C\left[\left(\frac{1-f}{1-R}\right)\Delta K\right]^{n} \frac{\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^{p}}{\left(1-\frac{K_{max}}{K_{1c}}\right)^{q}} \qquad [1]$$

$$f = \frac{Kop}{K\max} = A_0 + A_1 R \qquad if\ -2 \le R < 0$$

$$A_0 = (0.825 - 0.34\alpha + 0.05\alpha^2)\left[\cos(\pi S_{max}/2\sigma_{op})\right]^{1/\alpha} \qquad [2,3]$$

$$A_1 = (0.415 - 0.071\alpha)\ \ \sigma_{max}/\sigma_{op}$$

$$\Delta K_{th} = \Delta K_1 \left(\frac{1-R}{1-f(R)}\right)^{\left(1-RC_{th}^{m}\right)} \Big/ (1-A_0)^{\left(C_{th}^{p}-RC_{th}^{m}\right)} \qquad if\ R < 0 \qquad [4]$$

da/dN Integration

The Nasgro differential equation as defined in report D.6.2a by PoliMi formally can be written as

$$da / dN = C \cdot \Psi(a, \Delta\sigma)$$

and can be separated

$$da / \Psi(a, \Delta\sigma) = C \cdot dN .$$

The left hand side has to be integrated numerically from a_0 to a_{final} whereas the right hand side simply is $C \cdot N_i$ for the i-th block of the load spectrum with constant stress range $\Delta\sigma$.

a_{final} has to altered so that the (numerical) integral exactly matches $C \cdot dN$. This is best done by a root finder.

The numerical challenge is that one has a numerical procedure (root finder) with another numerical procedure (integration) in its argument (hierarchy of precisions).

Exception Handling

(In the Nasgro eqn's many exceptions are lurking.)

- If the stress intensity $\Delta K(a, \Delta\sigma)$ is $\leq \Delta K_{th}$ there is no crack growth; the integrand would become singular. Check for each load-block.
- $K_{max}(a, \sigma_{max})$ must be < K_C (fracture toughness); this would make the integrand infinite (crack instability); ruled out by limiting critical crack depth a_{crit} but still can be a fatal error.
- The geometry function is valid up to crack depth = Diameter/2; ruled out as above but still can be a fatal error.
- σ_{max} must be < $\sigma_{y,cyc0.2}$ which is 0.2% cyclic proof stress; ruled out in the blocks representing the load spectrum.
- The Newmann closure function can get negative if σ_{max} is too large; ruled out as above but still can be a fatal error.
- Computation of starting solution for root finder can fail as can root finder itself; these are fatal errors.

Input Data for A1N Steel

	Distribution	Mean (or value if fixed)	STDV or COV%
Diameter of the axle	Constant	0.160 [m]	
ΔK_{th}	Normal	Eq.[4] [MPam$^{1/2}$]	t.b.d.
Initial crack depth a_0	Rayleigh or Exponential	t.b.d. [m]	t.b.d.
Critical crack depth a_{crit}	Constant	t.b.d.; e.g. Diameter/4 [m]	
Log10(C) for ΔK in MPam$^{1/2}$ and da/dN in m/cycle	Normal (Mean from Table 2 of D.6.2a: Compression Precraking)	Log10(1.2555×10^{-11}) = -10.90	10% to 15%
Paris law exponent n	Constant	2.9457	
Nasgro exponent p	Constant	0.41	
Nasgro exponent q	Constant	0.41	
Load ratio, R	Constant	-1	
Constraint factor, α	Constant	2.50	
0.2% cyclic proof stress σ_y Mpa	Constant	357	
Threshold parameter ΔK_1	Constant (Table 1 in D.6.2a, CPLR)	3.064 [MPam$^{1/2}$]	
Threshold parameter C_{th}^p	Constant (Table 1 in D.6.2a, CPLR)	1.3406	
Threshold parameter C_{th}^m	Constant (Table 1 in D.6.2a, CPLR)	-0.0381	
POD-Model	Take from WIDEM or newer results	from Wolaxim	t.b.d.
Detectable crack size a_{det}	Take from WIDEM or newer results	from Wolaxim [m]	t.b.d.

Probability of detection (POD)

- The POD curves were developed by regression and analysis of the UT signal data and expressed in terms of the coefficients obtained from the fits in MINITAB.
- The POD results were used to derive an equivalent probability density function for the detectable flaw size (a_{det}) assuming a lognormal distribution with the two parameters expressed in terms of the coefficients obtained from the MINITAB regression analysis.
- The two lognormal distribution parameters are expressed in terms of the regression coefficients β_0, β_1, the scatter/scale parameter (POD_sig) and the signal threshold (signl_th):

$$\text{Par\_1} = e^{(-\beta_0 + \text{sgnl\_th})/\beta_1} \qquad \text{Par\_2} = \text{POD\_sig}/\beta_1$$

- In the probabilistic analysis, Par_1 is the median of a_{det} while Par_2 is the standard deviation of the natural logarithm of a_{det} (ie ln (a_{det})).

Load Spectrum Representation

No.	Ncycles	Smax
1	157 757	13.d0
2	1 722	72.d0
3	1	171.d0
4	40 050	41.d0
5	10	115.d0
6	200 000	27.d0
7	38	97.d0
8	157 757	13.d0
9	227	84.d0
10	7 950	59.d0
11	3	135.d0
12	40 050	41.d0
13	38	97.d0
14	200 000	27.d0
15	1 722	72.d0
16	21	135.d0
17	7 950	59.d0
18	10	115.d0
19	227	84.d0

Sum = 815 535 cycles

Example of Crack Growth Computation

1000 repetitions of these 19 blocks yields 8.16x108 ≈ **10^9 cycles or 2.5 Mio km.**

```
-----------------------------------------------------------------------------
  Newman clos.funct.: alfa=   2.500 and 0.2% cyclic proof stress= 357.000
  Stress ratio R=  -1.000
  Delta_K_1=    2.8327 Cp_th=    1.3406 Cm_th=   -0.0381
  da/DN eq.: C=  1.2555E-11 n=   2.946 p=   0.410 q=   0.410
  Diameter[m]=   0.160;  K_1c[MPa sqrt(m)]= 100.000;  a0[m]=5.0E-03 (i.e. 5 mm)
  -----------------------------------------------------------------------
  Input: Smax=  13.000 yields(R=-1) Smin= -13.000 and delta_sigma=  26.000
  Info: For above Smax and Azero=  0.0050 DK=   2.095 was < DK_th=  11.744 !
  -----------------------------------------------------------------------
  Input: Smax=  72.000 yields(R=-1) Smin= -72.000 and delta_sigma= 144.000
  Number of cycles =    1722; unscaled target value for Integral(a)=  2.1620E-08
  YRoot: Solution for a =  0.0050005458  Icount(cumul.)=     188
  -----------------------------------------------------------------------
  ………………………
  -----------------------------------------------------------------------
  Input: Smax=  84.000 yields(R=-1) Smin= -84.000 and delta_sigma= 168.000
  Number of cycles =     227; unscaled target value for Integral(a)=  2.8500E-09
  YRoot: Solution for a =  0.0116837834  Icount(cumul.)=     188
  -----------------------------------------------------------------------
  After repetition no.: 1000  value for a =  0.0116837834 (i.e. 11.7 mm)
  CPU-time after 1000 repetitions:  1.34 seconds
  Total calls of integrand:  2 389 309
```

Failure & Observation Events

The Failure event is described by a state function for fatigue failure

$$\{F\} : \{a - a_{crit} \leq 0\}$$

where *a* is crack depth after design life of axle and
a_{crit} the crack depth considered as critical.

The observation event (B) is described by a state function associated with the NDT capability of detection and size of flaws

$$\{B\} : \{a - (a_{det} + \varepsilon) \leq 0\} \quad or \quad \{a = (a_{det} + \varepsilon)\}$$

where a = crack depth at inspection and
a_{det} = minimum crack depth that a NDT technique can detect.

Conditional Probability

Like in WIDEM, the probability of failure of the axle due to fatigue loading can be under the condition of results from an NDT inspection.

The conditional probability can be written as:

$$P_{f,cond} = P\{F|B\} = \frac{P\{F \cap B\}}{P\{B\}}$$

The Strurel program **Sysrel** can perform this computation.
The main numerical effort is computation of the intersection probability in the numerator.

Application to:

- Existing Structures
- Proof Loading
- Quality Control

Task: **Reliability estimation given observations**

$$P\left(F \middle| B\right) = \frac{P\left(F \cap B\right)}{P\left(B\right)}$$

F: Failure Event **B: Observation**

$$F = \left\{ g\left(X\right) \leq 0 \right\}$$

$$B = \begin{cases} I = \left\{ h\left(X\right) \leq 0 \right\} \\ E = \left\{ e\left(X\right) = 0 \right\} \end{cases}$$

Example: Proof load test of a pipeline

Failure Criterion

$$F = \left\{ K_r - \frac{L_r}{\left[8/\pi^2 \cdot \ln(\sec(\pi \cdot L_r / 2)) \right]} \leq 0 \right\}$$

Simplified R6-rule

$$K_r = \sigma \sqrt{\pi a} Y(a) / K_{1c}; \;\; \sigma = p_i \;\; r/t; \;\; K_{1c} = fracture \;\; toughness$$

$$a = crack \;\; radius; \;\; Y(a) \approx 1.12 = geometry \;\; factor$$

$$L_r = p_i / p_0; \;\; p_i = inner \;\; pressure$$

$$p_0 = \left[1.07 \frac{t}{r} \frac{(R_y + R_m)}{2} \right] \left[1 - \frac{\pi a^2}{4t(a+t)} \right]$$

$$R_y = yield \;\; stress; \;\; R_m = rupture \;\; strength$$

$$t = wall \;\; thickness; \;\; r = cylinder \;\; radius$$

Crack growth during proof load test

$$\Delta a = \frac{\pi}{6(R_y K_{1c})^2} \frac{K_{1q}^4}{1 - \left(\frac{K_{1q}}{K_{1c}}\right)^2}$$

$$K_{1q} = \sigma_q \sqrt{\pi a} = stress \;\; \text{int}\, ensity$$

$$\sigma_q = qr / t = radial \;\; stress$$

Failure during proof load test

$$D = \left\{ K_r - \frac{L_r}{\left[8/\pi^2 \cdot \ln(\sec(\pi \cdot L_r / 2)) \right]} \leq 0 \,\middle|\, p_i = q, a = a + \Delta a \right\};$$

$$q = proof \;\; load \;\; (without \;\; error \;\; term)$$

Failure probability during proof load test

$$P_{proof\ load} = P(D)$$

Failure probability during service

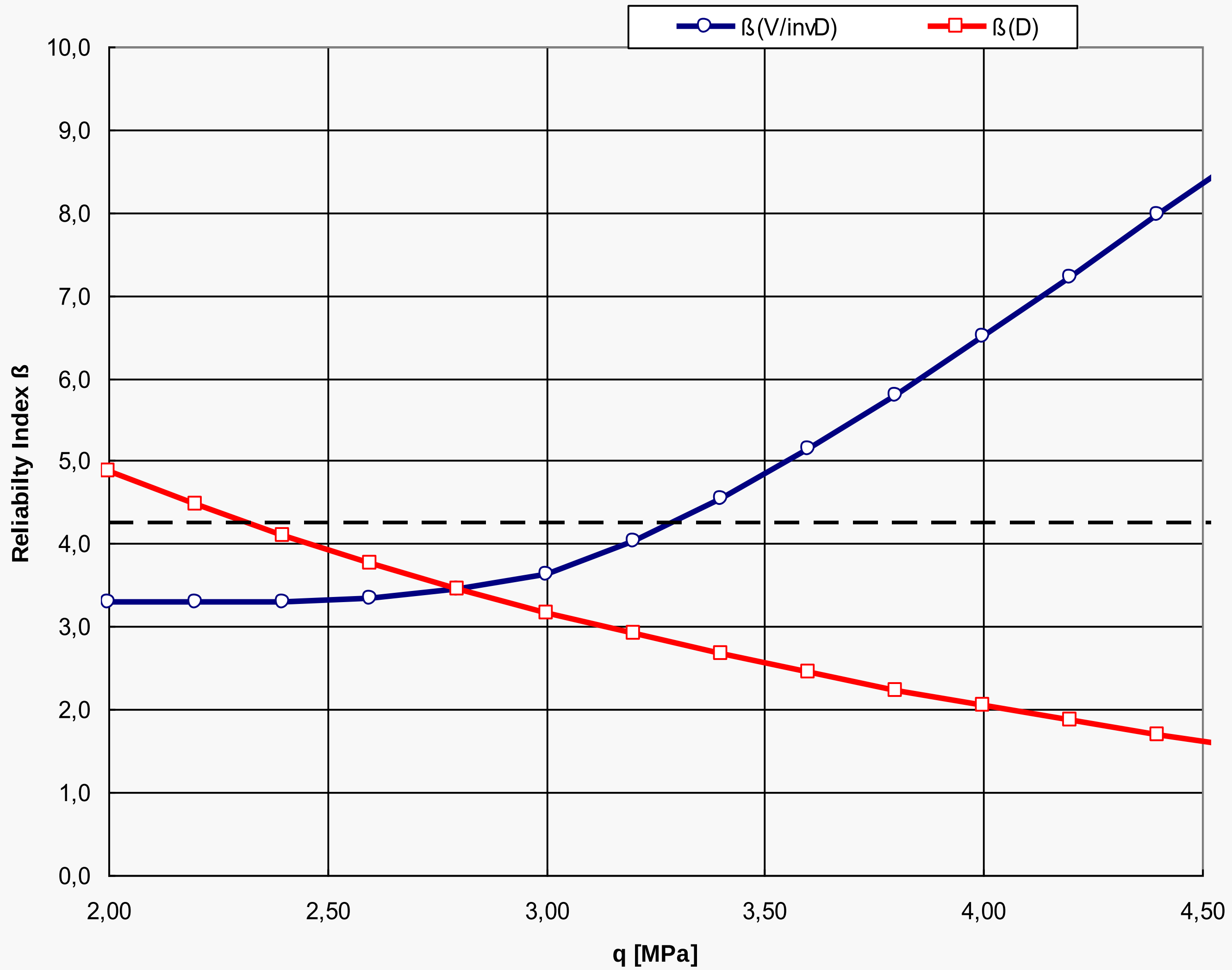
$$P(F|\bar{D}) = \frac{P(F \cap \bar{D})}{P(\bar{D})}$$

Target failure probability: 1×10^{-5} or $\beta = 4.265$

Stochastic model

Variable	Distr. function	Mean	C.o.V
p_i	Normal	2.5 [MPa]	0.10
R_y	Lognormal	1450 [MPa]	0.05
R_m	Normal	1700 [MPa]	0.05
K_{1c}	Lognormal	105 [MPa√m]	0.15
a	Rayleigh	1 [mm]	0.52
t	Normal	8.5 [mm]	0.02
r	Constant	600 [mm]	-

$$\rho(R_y, R_m) = \rho(R_y, K_{1c}) = \rho(R_m, K_{1c}) = -0.3$$


ß(V/invD)
ß(D)
10,0
9,0
8,0
7,0
6,0
5,0
4,0
3,0
2,0
1,0
0,0
Reliabilty Index ß
2,00
2,50
3,00
3,50
4,00
4,50
q [MPa]

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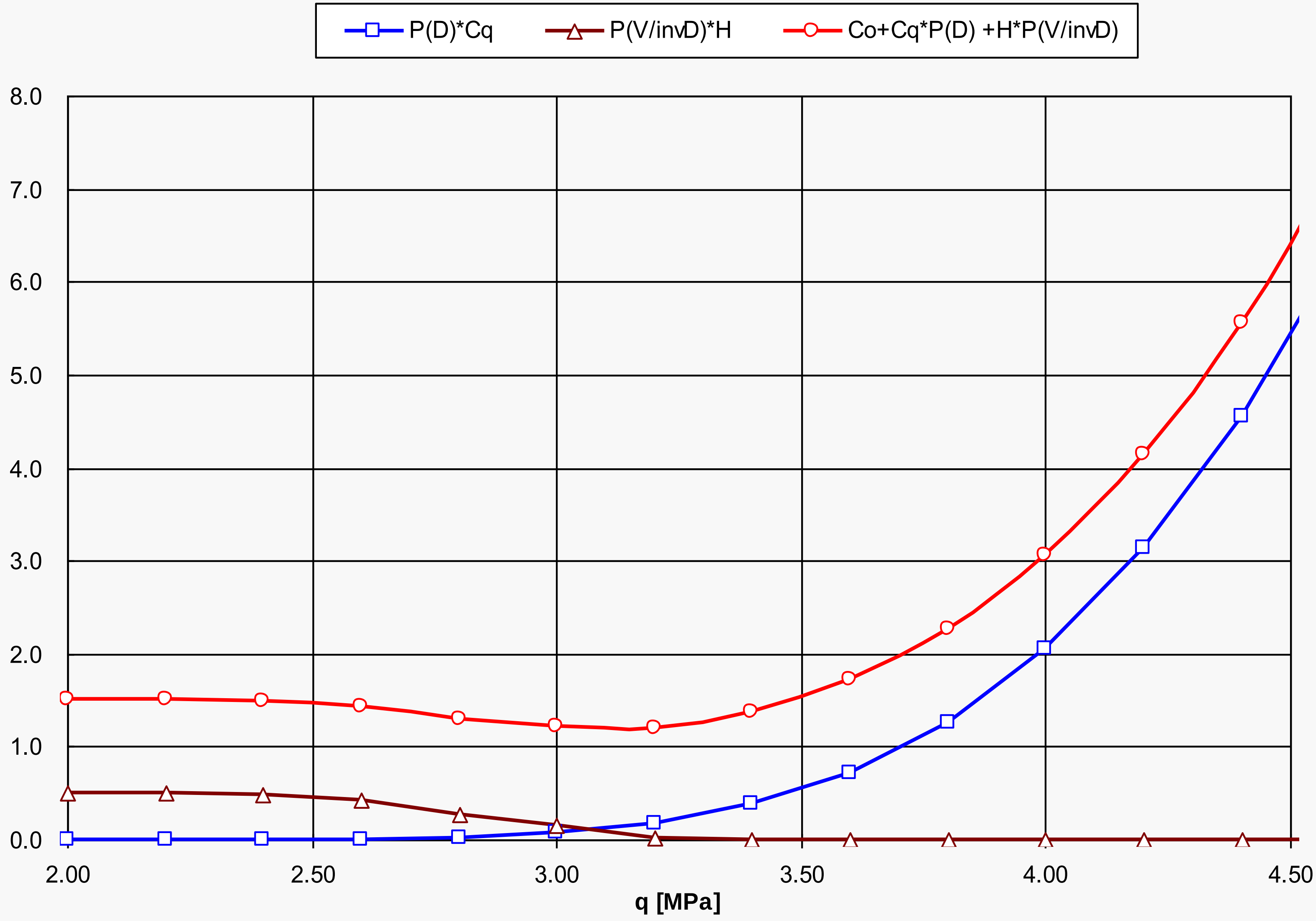
Cost Benefit Analysis

$$E[C] = C_0 + C_q P(D) + H \;\; P(F|\bar{D})$$

C_0 = 1: Manufacturing Cost

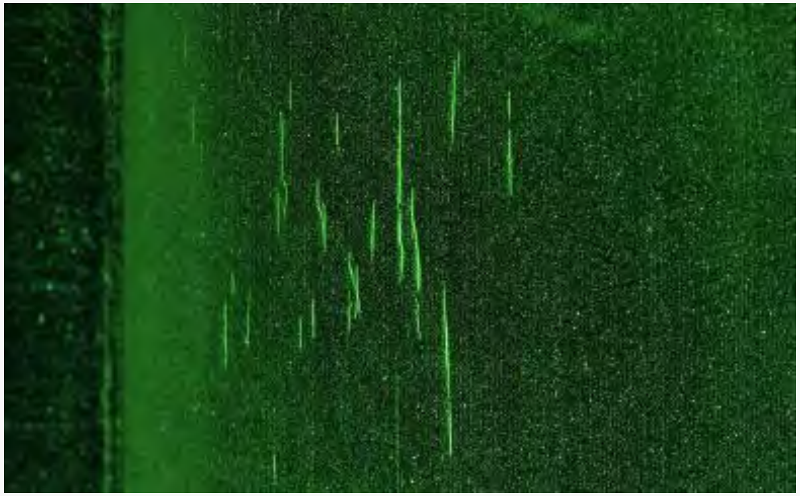
C_q = 100: Failure Cost during Proof Load

H = 1000: Failure Cost during Service


P(D)*Cq
P(V/invD)*H
Co+Cq*P(D) +H*P(V/invD)
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
2.00
2.50
3.00
3.50
4.00
4.50
q [MPa]

Probabilistic Life Extension Analysis

Recent axle failures show this kind of ‘surface cracking’ pattern



‘Life extension’ analysis is now possible with new computational tool

- What is the ‘failure probability’ of this axle under a given service scenario;
- What is the ‘increased risk’ respect to a new axle ?

Last Slide